Area Law in de Sitter Spacetime with Topological Soliton

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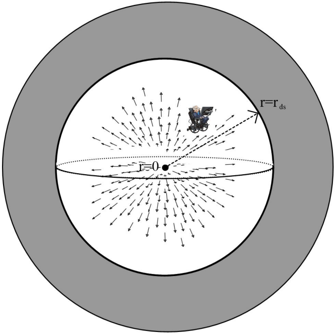
[1] Goal

We want to examine the area law
for the general-dimensional de Sitter space time
deformed by a nontrivial matter source,
and estimate the change of the entropy.

- *What kind of matter source?*
- *How do we calculate?*

[2] How

- *like this....*



[3] Black Hole Thermodynamics - Historical Review

- **In 1972, Bekenstein** proposed that the black hole area is proportional to the black hole entropy.

$$S_{\rm BH} = \frac{\ln 2}{2} \frac{A_{\rm BH}}{4G}$$

- **In 1973, Bardeen, Carter, and Hawking** suggested four laws of black hole thermodynamics.

  1. 0th : Constant κ (Constant T)
  2. 1st : $dM = \frac{\kappa}{8\pi G} dA + \Omega_{\rm H} dJ_{\rm H}$ $(dE = TdS)$ $\rightarrow T?\ S?$ Classically $T = 0$
  3. 2nd : $dA \geq 0$ $(dS \geq 0)$
  4. 3rd : $\kappa \not\rightarrow 0$ by a finite sequence of operations.

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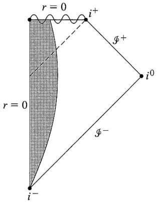
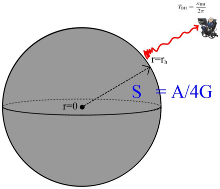
[4] Black Hole Thermodynamics - Historical Review

- **In 1975, Hawking** fixed the proportionality between T and κ by using the quantum field theory in the curved spacetime.



$$T_{\rm BH} = \frac{\kappa\big|_{r=r_{\rm h}}}{2\pi}$$

- **Area law (Bekenstein-Hawking Entropy)** :



$$\mathrm{d}E = \frac{\kappa}{8\pi G}\mathrm{d}A \to \frac{\kappa}{2\pi}\mathrm{d}\left(\frac{A}{4G}\right) \Leftrightarrow T\mathrm{d}S$$

$$\boxed{S_{\rm BH} = \frac{A_{\rm BH}}{4G}}$$

[5] Area Law?

Area Law :

$$S = \frac{k_B c^3}{\hbar} \frac{A}{4G}$$

- **Quantum gravitational equation**: equation with $\hbar$, G
- **Entoropy**$\sim$ **d.o.f.**: quantum gravitational states?
- **Area dependence** (not volume) $\rightarrow$ holographic principle
- **Problem of universality** :

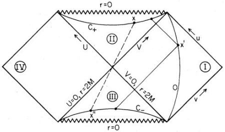
  different approaches to QG (string theory, LQG, induced gravity, $\cdots$

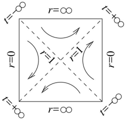
  different microstates

  $\rightarrow$ but same area law : why this result is universal?
  (check with nontrivial matter source?)
- **Information loss paradox** : thermal radiation, evolution to mixed states. violates unitarity of evolution, forbidden in ordinary QM.

[6] Black Hole Thermodynamics Extension (de Sitter spacetime)

- In 1976, Gibbons and Hawking extended the area law to the cosmological horizon (the event horizon in the de Sitter spacetime).

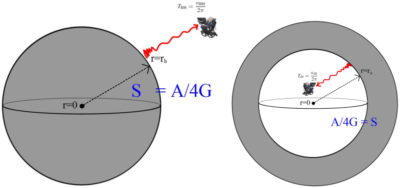




$$\mathrm{d}(-E) = \frac{\kappa_{\mathrm{dS}}}{8\pi G}\mathrm{d}A_{\mathrm{dS}}$$

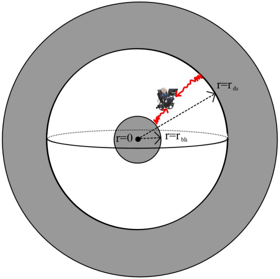
$$T_{\mathrm{dS}} = \frac{\kappa_{\mathrm{dS}}}{2\pi} \longrightarrow S_{\mathrm{dS}} = \frac{A_{\mathrm{dS}}}{4G}$$

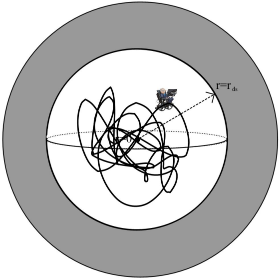
- The entropy accounts for the hidden information behind the horizon. In the dS spacetime, we see the hidden d.o.f. at the same time.



[7] Black Hole Thermodynamics Extension (SdS spacetime)

- Gibbons-Hawking : Controlled the thermodynamic variables with the simplest object → BH. with global hairs.
- Then, how about controlling more complicating matter source which is not hidden behind the black hole horizon? Can we see how the entropy behaves?
- If it deforms the geometry from the SdS or dS, the area law still holds?





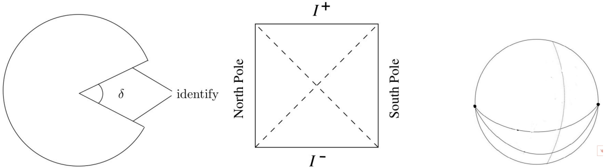
[8] Ex. Matter Distribution Without Horizon (SdS^3)

- 3 dimensional Schwarzschild de Sitter space (Spradlin 2001) :

$$ds^2 = -(1-8GE-r^2)dt^2 + \frac{1}{(1-8GE-r^2)}dr^2 + r^2 d\phi^2$$

$$T_{\mathrm{SdS}^3} = \frac{\sqrt{1-8GE}}{2\pi}, \quad S_{\mathrm{SdS}^3} = \frac{A^{\mathrm{H}}_{\mathrm{SdS}^3}}{4G} = \frac{\pi}{2G}\sqrt{1-8GE}, \quad (l \equiv 1)$$

- Same degrees of freedom in the gravity/matter side
- Localized matter at r=0 $\rightarrow$ a point-like source rather than a horizon
- Then the matter affects on the area law as a global effect (deficit angle)
  Tractable matter distribution without horizon $\rightarrow$ matter with deficit angle



[9] Our Model

- First, we will consider a matter which energy density goes as $1/r^2$ which is the maximum order we can consider as a field theory model.

$$\rho \longrightarrow \frac{1}{r^2}$$

- Even though the field energy is divergent when the radius goes to infinity, it will not change the background's vacuum dominant behavior.

$$\text{Energy density behavior : } \{\Lambda, -T^t{}_t\} \xrightarrow{r \to r_{\rm H} \gg 1} \{\Lambda \gg (d-2)\frac{v^2}{2r^2}\}$$

- For this consideration, let's choose a proper field configuration.
- proper field candidate : $O(N-1)$ symmetry, a hedge hog shape.

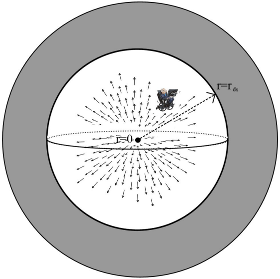
$$\phi^i = \hat{r}^i \phi(r), \quad (i = 1, \cdots, d-2)$$

$\longrightarrow$ This leads to same energy behavior.
$\longrightarrow$ This scalar field will have divergent energy when r goes to infinity. Since this is not the finite energy case, there exists a topological soliton solution even in the higher dimension (Derrick-Hobart theorem).

[10] Our Model

- Then how about the entropy changes from this deformation by the topological soliton?

$$\phi^i = \hat{r}^i \phi(r), \quad (i = 1, \cdots, d-2)$$



- Boundary condition:

$$\phi(0) = 0,\ \phi(r_{\rm h}) = v,\ M(0) = 0,\ \Omega(r_{\rm h}) = 1$$

[11] Assumptions in our Model

1. Dimension : $d > 3$

2. Gravity theory : minimal, Einstein-Hilbert action with a positive cosmological constant

$$S_{\rm EH} = \int d^d x \sqrt{-g}\,(R - 2\Lambda)$$

3. Matter source : spherically symmetric static scalar field

$$\phi^i \equiv \hat{\phi}^i \phi,\ \hat{\phi}^i \hat{\phi}^i = 1,\ O(d-1) \Rightarrow\ \phi^i = \hat{r}^i \phi(r), \quad (i = 1, \cdots, d-1)$$

4. Field potential : Higgs potential which is chosen in a minimal shape for supporting static global topological defect

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$

[12] Our Model (Action and Metric Ansatz)

- Action

$$S = \int d^d x \sqrt{-g} \left[\frac{1}{16\pi G}(R - 2\Lambda) - \left(\frac{g^{\mu\nu}}{2} \partial_\mu \phi^i \partial_\nu \phi^i + V(\phi) \right) \right]$$

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - v^2)^2$$

- Metric in the static coordinate

$$ds^2 = -e^{2\Omega(r)} A(r) dt^2 + \frac{dr^2}{A(r)} + r^2 d\Omega_{d-2}^2$$

where

$$d\Omega_{d-2}^2 = d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \cdots \sin^2\theta_{d-3} d\theta_{d-2}^2$$

$$A(r) \equiv 1 - \Delta_{\rm dS} - \left(\frac{r}{l}\right)^2 = 1 - \frac{2(\#)GM(r)}{r^{d-3}} - \left(\frac{r}{l}\right)^2$$

$$\Delta_{\rm dS} = \frac{16\pi GM(r)}{(d-2)\Omega_{d-2} r^{d-3}}, \quad (\#) = \frac{8\pi}{(d-2)\Omega_{d-2}}$$

[13] Equations of Motion and our Strategy

The equations of motion is given by

$$A\phi'' + A\phi'\left[\ln(r^{d-2}Ae^{\Omega})\right]' - \frac{d-2}{r^2}\phi = \frac{dV}{d\phi} = \lambda\phi(\phi^2 - v^2)$$
$$\frac{d-2}{r^{d-2}}\left[r^{d-3}(1-A)\right]' - 2\Lambda = 8\pi G\left[\frac{d-2}{r^2}\phi^2 + A(\phi')^2 + 2V\right]$$
$$\frac{d-2}{r}\Omega' = 8\pi G(\phi')^2$$

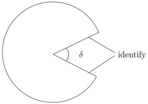
By using the asymptotic solution and the first law of thermodynamics, we will derive the entropy of the deformed system.

$$d(-E_{\delta\mathrm{dS}}) + P_{\delta\mathrm{dS}}d(-V_{\delta\mathrm{dS}}) = T_{\delta\mathrm{dS}}dS_{\delta\mathrm{dS}} \ \rightarrow \ S_{\delta\mathrm{dS}} = \frac{A^{\mathrm{H}}_{\delta\mathrm{dS}}}{4G_d}$$

Note that since the system has the pressure, we should consider PdV term. [Padmanabhan, 2002].

[14] Results from Solutions

- The geometry near the origin is Minkowski space time
- No deficit angle by the mild energy configuration



- Near the horizon → A deficit angle Δ_{deficit} ($\mathrm{dS}_d \to \delta\mathrm{dS}$),

$$\Delta_{\text{deficit}} = \Omega_{d-2}\left(1-(1-\delta)^{\frac{d-2}{2}}\right) \quad \left(\approx \Omega_{d-2}\frac{d-2}{2}\delta + \mathcal{O}(\delta^2)\right) \text{ for small } \delta \ll 1)$$

($\delta = 8\pi G v^2/(d-3)$, the positive deficit angle grows as v^2)

- Horizon radius : $r'_{\mathrm{H}} = l \;\rightarrow\; r_{\mathrm{H}} = \sqrt{1-\delta}\, l = \sqrt{1-\dfrac{8\pi G v^2}{d-3}}\, l$
- Temperature : $T_{\delta\mathrm{dS}} = \dfrac{\kappa}{2\pi} = \dfrac{\sqrt{1-\delta}}{2\pi l}$

[15] Entropy Calculation with dS boundary condition

As in the previous points, we will calculate the entropy as,

$$S_{\delta \mathrm{dS}} = \Delta S_{\delta \mathrm{dS}} + S_{\mathrm{dS}}$$

where

$$S_{\mathrm{dS}} = \frac{A^{\mathrm{H}}_{\mathrm{dS}}}{4G} = \frac{l^{d-2}\Omega_{d-2}}{4G}$$

From $d(-E) + Pd(-V) = TdS$, we get $\Delta S_{\delta \mathrm{dS}}$ as,

$$E_{\delta \mathrm{dS}} \approx \Omega_{d-2}\frac{d-2}{d-3}\frac{v^2}{2}r_{\mathrm{H}}^{d-3} = \Omega_{d-2}\frac{d-2}{16\pi G}l^{d-3}\delta(1-\delta)^{\frac{d-3}{2}}$$

$$P_{\delta \mathrm{dS}} = T^r{}_r \approx -\frac{d-2}{2}\frac{v^2}{r^2}, \quad P_{\delta \mathrm{dS}}\mathrm{d}(-V_{\delta \mathrm{dS}}) = (d-2)\frac{v^2}{2r_{\mathrm{h}}^2}\Omega_{d-2}r_{\mathrm{h}}^{d-2}\mathrm{d}r_{\mathrm{h}}$$

$$\boxed{\Delta S_{\delta \mathrm{dS}} = \frac{A_{\mathrm{dS}}}{4G}\left(-\frac{d-2}{2}\right)(1-\delta)^{\frac{d-4}{2}}\mathrm{d}\delta}$$

[16] Result : Area Law for the Distorted dS w/ Topological Defects

Then the entropy for the deformed system is given by

$$S_{\delta \mathrm{dS}} = S_{\mathrm{dS}} + \Delta S_{\delta \mathrm{dS}} = S_{\mathrm{dS}} + \int_{S(\delta=0)}^{S(\delta)} dS_{\delta \mathrm{dS}}$$
$$= \frac{A^{\mathrm{h}}_{\mathrm{dS}}}{4G}(1-\delta)^{\frac{d-2}{2}} \tag{11}$$

$$\boxed{S_{\delta \mathrm{dS}} = \frac{A^{\mathrm{h}}_{\delta \mathrm{dS}}}{4G} = \frac{1}{4G}\ell^{d-2}\Omega_{d-2}(1-\delta)^{\frac{d-2}{2}}}$$

Therefore, the area law still holds in the deformed system.

$$\delta \uparrow \Rightarrow \; r_{\mathrm{h}} \downarrow, \; T \downarrow, \; E \uparrow, \; P \uparrow, \; S \downarrow$$

As we expected, putting the non-trivial matter distribution leads the negative contribution to the entropy and in the case of the topological soliton the entropy changes with a factor of the solid deficit angle.

Conclusion

- When $\Lambda \neq 0$, especially, when $\Lambda > 0$, in the general dimensional spacetime, by adding a nontrivial matter source, we examined the entropy change.
- Since we have the non-trivial matter distribution example which has the exact expression for the entropy behavior in the classical(or semi-classical level), we could investigate more about its quantum origin in the subsequent research.